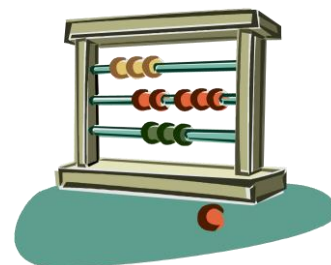

MULTIPLYING AND DIVIDING SIGNED NUMBERS

You may have learned how to solve an equation like $7n = 35$: You divide each side of the equation by 7,

$$\frac{7n}{7} = \frac{35}{7},$$

and conclude that $n = 5$. What if we come across an equation like $2x = -14$?

Our instinct should be to divide each side of the equation by 2 in order to isolate the x . But now we're faced with dividing a negative number (the -14) by a positive number (the 2). Not to worry — we'll figure it out.



❑ **MULTIPLYING SIGNED NUMBERS**

Recall that the result of multiplying two numbers is called the **product** of the two numbers. So, for instance, the product of 3 and 5 is 15. Some ways to represent the product of 3 and 5 are

$$3 \cdot 5 \quad 3 \times 5 \quad 3(5) \quad (3)5 \quad (3)(5) \quad \text{All these products equal 15}$$

Instead of using parentheses to indicate multiplication, square brackets are sometimes used. So $[3][5] = 15$ also.

Recall the **commutative property of multiplication**, where the order in which you

With variables, it's even easier: The product of 7 and x can be written **$7x$** , and the product of a and b can be written **ab** .

multiply a pair of numbers makes no difference in the product: $ab = ba$ for any numbers a and b .

Also recall the ***associative property of multiplication***, where parentheses can be “shifted” without changing the answer:

$$(ab)c = a(bc) \text{ for any numbers } a, b, \text{ and } c$$

Positive Times Positive

What should 6×4 be? Luckily, the answer is exactly what you would expect: **24**. In other words, what we learned as kids still holds: **A positive number times a positive number is positive.**

Positive Times Negative

This one’s not so obvious. Consider the multiplication problem $(7)(-1)$. We’ll determine the answer by sneaking up on the problem while it’s not looking:

$(7)(2) = 14$	since a positive times a positive is positive
$(7)(1) = 7$	same rule – or – anything times 1 is itself
$(7)(0) = 0$	anything times 0 is 0
$(7)(-1) = ???$	what should the product be?

What comes next in the sequence of products: 14, 7, 0, ? Since this sequence of numbers is decreasing by 7 at each step, the next number in the sequence must be -7 , and we see that $(7)(-1) = -7$. It appears that **a positive number times a negative number is negative.**

Negative Times Positive

To calculate $(-3)(4)$, first reverse the order of the factors (commutative property): $(-3)(4) = (4)(-3)$, which is now a product of a positive with a negative. By the previous rule, we know the



answer is -12 . Thus, **a negative number times a positive number is negative.**

Negative Times Negative

Now for the most interesting situation, the product of two negative numbers — for example $(-5)(-1)$. We'll get a running start and see what emerges.

$$\begin{array}{ll} (-5)(2) = -10 & \text{since a negative times a positive is negative} \\ (-5)(1) = -5 & \text{same rule – or – anything times 1 is itself} \\ (-5)(0) = 0 & \text{anything times 0 is 0} \\ (-5)(-1) = ??? & \text{what should the product be?} \end{array}$$

What comes next in the sequence of answers $-10, -5, 0$? Since this sequence of numbers is increasing by 5 at each step, it follows that the next number is 5, giving us the result:

$$(-5)(-1) = 5$$

We've reached the inescapable conclusion that **a negative number times a negative number is positive!**

See if you can deduce the two rules in the following box from the four rules stated above.

Multiplying Signed Numbers:

If the signs are the same, the product is positive.

If the signs are different, the product is negative.

Homework

1. Find the product:

- | | | | |
|----------------|----------------|-------------------|------------------|
| a. $(17)(3)$ | b. $(-4)(7)$ | c. $3(-10)$ | d. $(-3)(-4)$ |
| e. $-7(-2)$ | f. $2(-7)$ | g. $-1(8)$ | h. $-1(-9)$ |
| i. $(1)(-134)$ | j. $(-765)(0)$ | k. -3×-4 | l. $7 \cdot -10$ |
| m. $(-18)(-2)$ | n. $7(-3)$ | o. $-8(7)$ | p. $3 \cdot -9$ |
| q. $(-2)(99)$ | r. $(-1)(-7)$ | s. $(7)(-10)$ | t. $(-4)(-5)$ |

2. Find the product:

- | | | | |
|-----------------|------------------|-------------------|-------------------|
| a. $(0.2)(0.3)$ | b. $(-0.1)(0.1)$ | c. $(2.1)(-3)$ | d. $(-0.3)(-0.4)$ |
| e. $(-0.3)(5)$ | f. $2(-1.77)$ | g. $(-0.1)(-0.2)$ | h. $17(-0.2)$ |

3. Find the product in reduced form:

- | | | | |
|---------------------------------------|--|--|-------------------------------------|
| a. $\frac{2}{3} \times \frac{1}{5}$ | b. $-\frac{2}{3} \cdot \frac{3}{4}$ | c. $\frac{4}{5} \left(-\frac{5}{4}\right)$ | d. $(-2) \left(-\frac{7}{2}\right)$ |
| e. $\frac{4}{5} \times -\frac{10}{3}$ | f. $\left[-\frac{1}{2}\right] \left[\frac{2}{99}\right]$ | g. $\left(-\frac{2}{3}\right) \left(-\frac{3}{2}\right)$ | h. $10 \left(-\frac{3}{40}\right)$ |

4. Find the product:

Example: $(-2)(3)(-4)$
 $= (-6)(-4)$ (multiply the first two factors)
 $= 24$

- | | | |
|-------------------------------|---------------------------|----------------------------|
| a. $(-2)(-3)(4)$ | b. $7 \times -6 \times 3$ | c. $(-1)(-2)(-3)(-4)$ |
| d. $4 \cdot 4 \cdot 4(-1)(3)$ | e. $5(-1)(-1)(-2)$ | f. $4(-3)(2)(-8)$ |
| g. $(-2)(-1)(-3)$ | h. $(-7)(6)(2)(-1)$ | i. $(-1)(-1)(-1)(8)$ |
| j. $(-1)(-3)(-5)(-7)$ | k. $-3(-4)(5)2$ | l. $(-2)(-2)(-1)(-3)(-5)0$ |

*****DIVIDING SIGNED NUMBERS*****

The secret to the division rules for signed numbers is the fact that division is checked by multiplication. For example,

$$\frac{56}{7} = 8, \text{ precisely because } 8 \times 7 = 56.$$

Positive Divided by Positive

$$\frac{6}{2} = 3, \text{ since } (3)(2) = 6.$$

Therefore, **a positive number divided by a positive number is positive.**

Negative Divided by Positive

$$\frac{-6}{2} = -3, \text{ because } (-3)(2) = -6.$$

Thus, **a negative number divided by a positive number is negative.**

Positive Divided by Negative

$$\frac{6}{-2} = -3, \text{ which is checked by seeing that } (-3)(-2) = 6.$$

Conclusion: **a positive number divided by a negative number is negative.**

Negative Divided by Negative

$$\frac{-6}{-2} = 3, \text{ which is confirmed by the fact that } (3)(-2) = -6.$$

We see that **a negative number divided by a negative number is positive.**

NOTES: 1. Do these four rules for dividing signed numbers remind you of anything? The rules for dividing signed numbers are the same as the rules for multiplying signed numbers. (Now, that's nice.)

2. Note that $\frac{-10}{2} = \frac{10}{-2} = -\frac{10}{2},$

because each of these three division problems has a

quotient of -5 . In general, the following three fractions are equal:

$$\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$$

Homework

5. Using the logic above, which explains how signed numbers are divided, explain why each division problem is correct:

a. $\frac{22}{11} = 2$

b. $\frac{-21}{7} = -3$

c. $\frac{88}{-22} = -4$

d. $\frac{-100}{-20} = 5$

Here's the bottom line for dividing signed numbers:

Dividing Signed Numbers:

If the signs are the same, the quotient is positive.

If the signs are different, the quotient is negative.

EXAMPLE 1: Find each quotient:

A. $\frac{45}{-9} = -5$ (positive divided by negative is negative)

B. $\frac{-250}{-25} = 10$ (negative divided by negative is positive)

C. $\frac{-21}{15} = -\frac{21}{15} = -\frac{7}{5}$ (negative divided by positive is negative)

D. $\frac{-8}{-12} = \frac{8}{12} = \frac{2}{3}$ (negative divided by negative is positive)

E. $\frac{18}{-7} = -\frac{18}{7}$ (positive divided by negative is negative)

F. $\frac{-4}{-9} = \frac{4}{9}$ (negative divided by negative is positive)

Homework

6. Find the quotient (leave answers as simplified fractions):

a. $\frac{30}{5}$	b. $\frac{60}{-10}$	c. $\frac{-100}{20}$	d. $\frac{-14}{-2}$
e. $\frac{3}{6}$	f. $\frac{-18}{4}$	g. $\frac{24}{-5}$	h. $\frac{-34}{-24}$
i. $\frac{-1}{9}$	j. $\frac{-345}{-345}$	k. $\frac{10}{-10}$	l. $\frac{-120}{-70}$

7. Evaluate each expression:

a. $\frac{5-1}{1-2}$	b. $\frac{10(-2)}{-4}$	c. $\frac{3^2-9}{7}$
d. $\frac{18-2}{2-18}$	e. $\frac{1+2+3}{-2-4}$	f. $\frac{3^2-1^3}{6-5}$

8. Evaluate each expression:

Example: $4 - 5[-3 + 2(3 - 6)]$

$$= 4 - 5[-3 + 2(-3)] \quad \text{(start with innermost parentheses)}$$

$$= 4 - 5[-3 - 6] \quad \text{(multiply before adding)}$$

$$= 4 - 5[-9] \quad \text{(now for the brackets)}$$

$$= 4 + 45 \quad \text{(multiply before subtracting)}$$

$$= \mathbf{49} \quad \text{(and we're done)}$$

a. $2[5 - 9(-2)]$

b. $-3[1 + 2(-5)]$

c. $4 + 3[2 - 3(4)]$

d. $4 - 5[1 + 5(1 - 4)]$

e. $10 - 8[2(3) - 4(2)]$

f. $7 - 2[8 - 2(3 - 4)]$

g. $4 - 7[-5 - (2 - 9)]$

h. $10 - 2[10 - 2(3 - 7)]$

Review Problems

9. A negative divided by a negative is positive. Explain why.

10. a. $(-5)(-2)(-3) =$ b. $\frac{45}{-9} =$ c. $\frac{-80}{25} =$

d. $\frac{-12}{-48} =$ e. $\frac{2-8-10}{12-1-5} =$

11. a. $(-5)(2)(-4) =$ b. $\frac{-45}{-9} =$ c. $\frac{80}{-15} =$

d. $\frac{-36}{-48} =$ e. $\frac{2-8+10}{12+1-5} =$

12. $4 - 3[2(-3) - 5(8 - 6)] =$

The entire chapter can be summarized like this:

For both multiplying and dividing,

Same signs $\Rightarrow$ Positive answer

Different signs $\Rightarrow$ Negative answer

Solutions

1. a. 51 b. -28 c. -30 d. 12 e. 14 f. -14
 g. -8 h. 9 i. -134 j. 0 k. 12 l. -70
 m. 36 n. -21 o. -56 p. -27 q. -198 r. 7
 s. -70 t. 20

2. a. 0.06 b. -0.01 c. -6.3 d. 0.12 e. -1.5
 f. -3.54
 g. 0.02 h. -3.4

3. a. $\frac{2}{15}$ b. $-\frac{1}{2}$ c. -1 d. 7 e. $-\frac{8}{3}$ f. $-\frac{1}{99}$
 g. 1 h. $-\frac{3}{4}$

4. a. 24 b. -126 c. 24 d. -192 e. -10 f. 192
 g. -6 h. 84 i. -8 j. 105 k. 120 l. 0

5. a. $(2)(11) = 22$ b. $(-3)(7) = -21$
 c. $(-4)(-22) = 88$ d. $(5)(-20) = -100$

6. a. 6 b. -6 c. -5 d. 7 e. $\frac{1}{2}$
 f. $-\frac{9}{2}$ g. $-\frac{24}{5}$ h. $\frac{17}{12}$ i. $-\frac{1}{9}$ j. 1
 k. -1 l. $\frac{12}{7}$

7. a. -4 b. 5 c. 0 d. -1 e. -1 f. 8

8. a. 46 b. 27 c. -26 d. 74
 e. 26 f. -13 g. -10 h. -26

9. $\frac{\text{neg}}{\text{neg}} = \text{pos}$ because when the division is checked by multiplication, we

confirm that $\text{pos} \times \text{neg} = \text{neg}$ ✓

10. a. -30 b. -5 c. $-\frac{16}{5}$ d. $\frac{1}{4}$ e. $-\frac{8}{3}$

11. a. 40 b. 5 c. $-\frac{16}{3}$ d. $\frac{3}{4}$ e. $\frac{1}{2}$

12. 52

“Let us think of education as the means
of developing our greatest abilities,
because in each of us there is a private
hope and dream which, fulfilled, can be
translated into benefit for everyone and
greater strength of the nation.”

– *John F. Kennedy*

